

EXPERIMENTAL STUDY OF HEAT TRANSFER  
IN MESHED MATRICES

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The article presents results of a study concerning metallic meshes of fine wire up to 30 μ in diameter. They are used in high-efficiency regenerators for gas refrigerators and they yield a compactness of up to 4 · 10<sup>4</sup> m<sup>2</sup>/m<sup>3</sup>.

Existing data on the heat transfer in such matrices [1-3] are most contradictory (Fig. 1). The test values given in [1] and [2] range between dashed lines 1, while the test values given in [3] range between dashed lines 2. A comparison of these results indicates a wide dispersion, more than by one order of magnitude at low and medium values of the Reynolds number. The matrices studied in [1, 2] were short, i. e., had a small H/D<sub>e</sub> ratio (for example, the number of meshes per packet in [1] did not exceed 40) while the matrices in [3] consisted of many densely packed meshes (300-790). Here H denotes the length (height) of a packet in the direction of the flowing stream and D<sub>e</sub> denotes the equivalent diameter of the matrix channels.

In order to explain this dispersion of values and to obtain reliable data, the authors had built an apparatus with which both the heat transfer and the resistance could be measured in matrices consisting of any number of packets, from one to several hundred, in a steady stream over a wide range of the Reynolds number. Steady flow was ensured by a continuous generation of heat within the matrix mass by alternating electric current. The heat was carried away by an air stream blowing on the matrix. With this stand we tested packets consisting of meshes in three sizes: No. 004, No. 0071, and No. 0112; we tested 19 models with different relative lengths.

Our results clearly indicate different heat transfer characteristics for matrices with different numbers of meshes per packet. In Fig. 1 are shown the data for six matrices with a No. 004 mesh (solid lines 3-8). As the number of meshes is increased or, more precisely, as the relative length H/D<sub>e</sub> of a matrix is increased, the heat transfer proceeds at a lower rate. The maximum heat transfer occurs with one mesh or two meshes per packet.

These data on heat transfer are generalized by using the Reynolds number Re and the relative length H/D<sub>e</sub>. The criterial equation of heat transfer in matrices with densely packed fine meshes is, for H/D<sub>e</sub> in the 2-210 range

$$Nu = 1.21Re^{0.47} \left(\frac{H}{D_e}\right)^{-\frac{0.8}{Re^{0.25}}} \tag{1}$$

and for H/D<sub>e</sub> > 210

$$Nu = 0.05Re^{0.85} \tag{2}$$

The Reynolds number in Eqs. (1) and (2) varies from 10 to 500.

The wide variation in the heat transfer rate following a change in

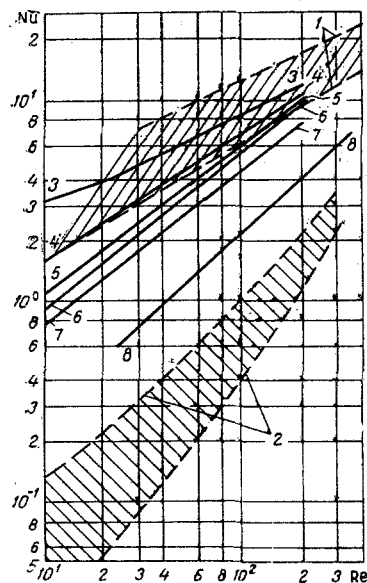


Fig. 1. Heat transfer in meshed matrices: 1) range of data in [1, 2]; 2) range of data in [3]. Our data for a mesh No. 004 with: 3-3) H/D<sub>e</sub> = 1.1-2.2; 4-4) 5.5; 5-5) 16.5; 6-6) 27.5; 7-7) 45; 8-8) 212.

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the relative length can be explained according to the model of a mesh matrix where the stream is treated as an internal flow along channels of complex shapes. Moreover, the mesh packing reduces sharply the turbulence intensity in the entering stream. One may assume, therefore, that increasing the number of meshes has a stabilizing effect on the stream.

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THERMOPHYSICAL METHOD OF MEASURING THE  
TEMPERATURE CHARACTERISTIC OF  
ELECTRICAL RESISTANCE IN METALS

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A method is proposed for measuring the temperature characteristic of electrical resistance in metals by heating a thin metallic filament with a constant electric current in an inert-gas atmosphere under known flow conditions. In such tests one measures the current through and the voltage drop across the filament, from which one then calculates the filament resistance and the electric power in the given heating mode. Two procedures have been developed for determining the temperature, both based on the laws of convective heat transfer at a filament in a stream under large temperature drops.

The first procedure is as follows. With the test specimen in a steady state, one determines the relative filament resistance  $R/R_0$  as a function of the electric power  $P$  at two different stream velocities and, on the basis of these data, one plots an  $R/R_0 = f(\kappa)$  curve with  $\kappa$  denoting the difference in electric power necessary to heat the filament to the same temperature at different stream velocities (different values of the convective heat transfer coefficient). The magnitude of  $\kappa$  does not depend on the emissivity of the material and, together with certain known criterial relations, it yields the filament temperature.

The second procedure involves the use of a reference filament with a known temperature-dependence of its electrical resistance. Having determined  $R/R_0$  as a function of  $\kappa$  for the test material and the temperature  $T$  as a function of  $\kappa_{\text{ref}}$  for the reference filament, one can compare the values  $\kappa$  and  $\kappa_{\text{ref}}$ , thus correlating the quantities  $R/R_0$  and  $T$  for the test material. In this method of determining the filament temperature one does not have to know the convective heat transfer coefficients nor have to measure velocities of the oncoming stream.

These procedures have been checked experimentally. The temperature characteristics of electrical resistance, which have been thus determined for copper, silver, and VR-5 alloy over a wide temperature range (approximately 1000°C), agree closely with published data.

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DETERMINATION OF THE MELTING TIME FOR A  
HEAT CARRIER IN A PIPE SEGMENT HEATED  
AT ONE END

B. A. Solov'ev

UDC 536.3

The heating and the melting of a liquid-metal heat carrier in a pipe with thermal insulation at one end and a heat source at the other end, and with heat radiating into the ambient medium, are described by a system of equations consisting of a nonlinear second-order differential equation and an integrodifferential equality with appropriate boundary conditions of the first and of the second kind. A conversion into dimensionless form has made it possible to reduce the number of independent variables, to solve the system of equations on a computer, and to present the results graphically in terms of relative melting time  $\bar{\tau} = \tau / (L^2 c \rho / \lambda)$  as a function of the pipe transmittivity parameter  $c_m = \epsilon_r \sigma \Pi T_m^3 L^2 / (\lambda F)$  and of the referred ambient temperature  $\theta_a = T_a / T_m$  — with the temperature at the hot end given either referred to the melting temperature  $\theta_{in} = T_{in} / T_m$  or to the relative length of the pipe segment  $\theta_{in}^1 = \partial \theta_{in} / \partial (x/L)$  (which corresponds to constant thermal input power to the hot end). The graphs plotted for potassium, sodium, and lithium can be used for solving various problems related to melting these heat carriers in pipes of various shapes. The use of these graphs is illustrated on several examples.

NOTATION

F is the cross section area;  
 $\Pi$  is the radiation perimeter;  
L is the pipe length;  
x is the space coordinate;  
 $T_m$  is the melting temperature;  
 $T_a$  is the radiation temperature of the ambient medium;  
 $T_{in}$  is the temperature at the hot end;  
 $\epsilon_r$  is the referred emissivity of the pipe surface;  
 $\rho$  is the density;  
c is the specific heat;  
 $\lambda$  is the thermal conductivity;  
 $\sigma$  is the Stefan-Boltzmann constant;  
 $\tau$  is the time.

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PULSE HEATING OF A PLATE WITH FINITE WIDTH  
SEPARATING TWO MEDIA

S. A. Drozdov and V. F. Salokhin

UDC 536.212

The temperature of a heat source is calculated theoretically; this heat source being located on the plane boundary between two semiinfinite media with different thermal properties  $(a_1, \epsilon_1)$ ,  $(a_2, \epsilon_2)$  and having the shape of a strip with finite width  $2l$  which at time  $t = 0$  is activated at a constant (in time) and uniform (over the surface) thermal flux density  $q$ . Such models come up in the development of pulse methods for thermophysical measurements with the use of thin-film resistance probes, and they are of interest for the calculation of thermal fields in microelectronic circuitry.

Disregarding the thickness and the specific heat of the source, the authors have obtained the following expression for its mean temperature as a function of time:

$$T(t) = \frac{2q\sqrt{t}}{(\epsilon_1 + \epsilon_2)\sqrt{\pi}} \left[ 1 - \frac{\sqrt{t}}{4l} \int_0^{2l/\sqrt{t}} \Psi_2(\xi) d\xi - \frac{l}{\sqrt{t}} \int_{2l/\sqrt{t}}^{\infty} \Psi_2(\xi) \frac{d\xi}{\xi^2} \right],$$

$$\Psi_2(x) = \left\{ \epsilon_1 \left[ \omega(i\alpha_1) - \frac{\epsilon_2}{\sqrt{\mu}} \omega\left(i\alpha_1 \frac{\epsilon_2}{\sqrt{\mu}}\right) \right] e^{-\alpha_1^2 x^2} - \epsilon_2 \left[ \omega(i\alpha_2) - \frac{\epsilon_1}{\sqrt{\mu}} \omega\left(i\alpha_2 \frac{\epsilon_1}{\sqrt{\mu}}\right) \right] e^{-\alpha_2^2 x^2} \right\},$$

where  $\omega(z) = e^{-z^2} [1 + 2i/\sqrt{\pi} \int_0^z e^{\xi^2} d\xi]$  is the complex error function and

$$\mu = \frac{a_1 \epsilon_1^2 - a_2 \epsilon_2^2}{\epsilon_1^2 - \epsilon_2^2}, \quad \alpha_n = \frac{x^2}{4a_n}, \quad n = 1, 2, \quad i^2 = -1.$$

Furthermore, the asymptotic behavior of this solution is examined in the case of short and long time periods. For  $t \rightarrow 0$ , specifically,

$$T(t) \approx \frac{2q\sqrt{t}}{(\epsilon_1 + \epsilon_2)\sqrt{\pi}} \left[ 1 - \frac{\sqrt{F_1}}{2\sqrt{\pi}} \Phi_1(k_a, k_e) + \dots \right],$$

$$\Phi_1 = \frac{1 - k_a k_e}{1 - k_e} - g(\gamma) \frac{k_e}{|1 - k_e|} \sqrt{\frac{|1 - k_a^2|}{|1 - k_e^2|}}, \quad g(\gamma) = \begin{cases} \arccos \gamma & \text{at } \gamma \leq 1, \\ -\text{Arch } \gamma & \text{at } \gamma > 1, \end{cases}$$

$$\gamma = \frac{k_a + k_e}{1 + k_a k_e}, \quad k_a^2 = \frac{a_2}{a_1}, \quad k_e = \frac{\epsilon_2}{\epsilon_1}.$$

The factor outside the brackets represents the solution to the one-dimensional version of the problem, where the width of the heat source increases infinitely; the expression inside the brackets determines, accordingly, the edge effects. This formula yields the validity criterion for the one-dimensional solution, namely

$$F_1 \ll 4\pi\Phi_1^2.$$

NOTATION

$a_n$  is the thermal diffusivity of a medium;  
 $\epsilon_n$  is the thermal activity of a medium;

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$t$  is the time;  
 $F_n = a_n t / l^2$  is the Fourier number.

INTERNAL HEAT TRANSFER IN A PLATE UNDER  
NONLINEAR BOUNDARY CONDITIONS

B. Ya. Lyubov and N. I. Yalovoi

UDC 536.3.536.25

The solution to the differential equation of heat conduction

$$\frac{\partial v}{\partial \tau} = \frac{\partial^2 v}{\partial x^2} \quad (1)$$

for a plate with a nonlinear boundary condition

$$\frac{\partial v}{\partial x} \Big|_{x=1} = F(v_{x=1}, \tau) \quad (2)$$

at the surface, with a symmetry

$$\frac{\partial v}{\partial x} \Big|_{x=0} = 0 \quad (3)$$

and with an arbitrary initial temperature distribution

$$v|_{\tau=0} = \dot{f}(x) \quad (4)$$

is found in the form

$$v(x, \tau) = \sum_{n=0}^{\infty} \frac{(1-x)^{2n}}{(2n)!} \varphi^{(n)}(\tau) + \sum_{n=0}^{\infty} \frac{(1-x)^{2n+1}}{(2n+1)!} \psi^{(n)}(\tau), \quad (5)$$

where

$$\psi(\tau) = -F[\varphi(\tau), \tau].$$

Function  $\varphi(\tau)$  is determined from the integral Volterra equation of the second kind. In the case of a parabolic initial temperature distribution, for example, we have

$$\varphi(\tau) = v_1 - 2\Delta v_0 \Phi_1(\tau) + \int_0^{\tau} F[\varphi(t), t] \Phi_2(\tau - t) dt, \quad (6)$$

where

$$\Phi_1(\tau) = \frac{1}{6} + 2 \sum_{k=1}^{\infty} \frac{(-1)^k}{\pi^2 k^2} \exp(-\pi^2 k^2 \tau),$$

$$\Phi_2(\tau) = 1 + 2 \sum_{k=1}^{\infty} \exp(-\pi^2 k^2 \tau).$$

With  $\varphi(\tau)$  determined from Eq. (6), it is simple to obtain the values of  $\varphi^{(n)}(\tau)$  and  $\psi^{(n)}(\tau)$  needed for calculating the temperature across a body section.

Two specific problems are analyzed on the basis of this theory: heating of a plate by thermal radiation and heating of a plate when the heat transfer coefficient varies as a function of time.

NOTATION

$$v(x, \tau) = T(x, \tau)/T_m;$$

$$\tau = at/R^2;$$

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$x = x/R$ ;  
 $T(x, \tau)$  is the temperature function of the plate;  
 $T_m$  is the temperature of the heat emitting medium (if this temperature varies as a function of time, then  $T_m$  is its maximum value);  
 $x$  is the space coordinate;  
 $2R$  is the plate thickness;  
 $a$  is the thermal diffusivity;  
 $t$  is the time;  
 $v_1$  is the surface temperature;  
 $\Delta v_0$  is the initial temperature drop across a plate section.



HEAT CONDUCTION WITH AN ARBITRARY PERIODIC  
 VARIATION OF THE HEAT TRANSFER COEFFICIENT  
 AND OF THE AMBIENT TEMPERATURE

B. Ya. Lyubov, N. I. Yalovoi,  
 and I. N. Manusov

UDC 536.2.01

The problem of heat conduction in a plate is analyzed for the case where both the heat transfer coefficient  $\alpha(\tau)$  and the ambient temperature  $T_a(\tau)$  are periodic functions of time. It is assumed that functions  $\alpha(\tau)$  and  $T_a(\tau)$  satisfy the Dirichlet condition.

The solution is obtained by a combined application of the operational method and successive approximations.

Simple harmonic fluctuations of the heat transfer coefficient and of the ambient temperature

$$\begin{aligned} Bi(Fo) &= \frac{a_0}{2} + a_1 \cos Pd Fo, \\ v_c(Fo) &= \frac{c_0}{2} + c_1 \cos Pd Fo \end{aligned} \quad (1)$$

are analyzed thoroughly here.

The calculation formulas for this case are

$$\begin{aligned} v_0(Fo, X) &= \frac{a_0}{2} \cdot \frac{c_0}{2} \Theta_0\left(\frac{X}{2}, Fo\right) + \left(a_1 \frac{c_0}{2} + c_1 \frac{a_0}{2}\right) \\ &\times \int_0^{Fo} \Theta_0\left(\frac{X}{2}, Fo-t\right) \cos Pd t dt + a_1 c_1 \int_0^{Fo} \Theta_0\left(\frac{x}{2}, Fo-t\right) \cos^2 Pd t dt, \end{aligned} \quad (2)$$

and

$$v_k(Fo, X) = -\frac{a_0}{2} \int_0^{Fo} \Theta_0\left(\frac{X}{2}, Fo-t\right) v_{k-1}(t, 1) dt - a_1 \int_0^{Fo} \Theta_0\left(\frac{X}{2}, Fo-t\right) v_{k-1}(t, 1) \cos Pd t dt, \quad (3)$$

where

$$\begin{aligned} k &= 1, 2, 3, \dots, \\ \Theta_0(z, t) &= 1 + 2 \sum_{k=1}^{\infty} (-1)^k \exp(-\pi^2 k^2 t) \cos 2\pi k z. \end{aligned}$$

Graphs representing approximations of the temperature function prove the convergence of this method, because the second approximation lies between the zeroth and the first one, the third approximation lies between the first and the second one, etc.

An analysis of expressions (2) and (3) shows that the process of successive approximations is sufficiently convergent only when  $a_0/2 < 1$  and  $a_1 < 1$ . In view of this, a modified solution is given here suitable for practical use within the  $a_0/2 > 1$  and  $a_1 > 1$  range.

The concept of this method can be easily extended to bodies with other shapes.

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## NOTATION

$V_a(Fo) = (T_a(Fo) - T_0)/(T_A - T_0);$	
$T_0$	is the initial plate temperature;
$T_A$	is the amplitude of fluctuations of ambient temperature;
$Fo = a\tau/R^2$	is the Fourier number;
$Bi(Fo) = \alpha(Fo)R/\lambda;$	
$Pd \cdot Fo$	is the dimensionless cycle time;
$Pd$	is the Predvoditelev number;
$X = x/R$	is the referred space coordinate.

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CALCULATION OF THE EIGENVALUES IN THE  
 PROBLEM OF HEAT TRANSFER DURING LAMINAR  
 FLOW THROUGH A CIRCULAR CHANNEL

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 and A. K. Fedyukovich

UDC 532.595

In [1] was given an analytical solution of the temperature field in a laminar stream of fluid flowing through a circular channel and heated externally by convection. In order to use this equation in engineering, one must know the numerical values of the characteristic roots  $\mu_n$ , the eigenfunctions  $\psi_n$ , and the constant coefficients  $A_n$  of the expansion. These values were given in [1] for only a very narrow range of the Biot number, however. For this reason, values of  $\mu_n$ ,  $\psi_n$ , and  $A_n$  have been subsequently calculated by computer for a much wider range of the Biot number. The results have been appropriately tabulated.

The eigenvalue and eigenfunction problem (the Sturm–Liouville problem) is to find a solution to the system of equations

$$\psi'' + \frac{\psi'}{R} + \mu^2(1 - R^2)\psi = 0, \quad (1)$$

$$\psi' = 0 \text{ at } R = 0, \quad (2)$$

$$\psi' = -\text{Bi} \psi \text{ at } R = 1. \quad (3)$$

The integral of this system is sought in the class of special functions. As has been shown in [2], the general solution to problem (1)–(3) can be written as

$$\psi = \exp\left(-\mu \frac{R^2}{2}\right) F_a(\alpha, \gamma, \mu R^2), \quad (4)$$

where  $F_a(\alpha, \gamma, \mu R^2)$  is a confluent hypergeometric function defined in terms of the infinite sum:

$$F_a(\alpha, \gamma, \mu R^2) = 1 + \frac{\alpha}{\gamma} \mu R^2 + \frac{\alpha(\alpha+1)}{\gamma(\gamma+1)} \frac{\mu^2 R^4}{2!} + \dots \quad (5)$$

In this case  $\alpha = 1/2 - \mu/4$  and  $\gamma = 1$ . Then Eq. (4) transforms into

$$\psi = \exp\left(-\mu \frac{R^2}{2}\right) \left[ 1 + \frac{(2-\mu)\mu}{4} R^2 + \frac{(2-\mu)(6-\mu)\mu^2}{16} \frac{R^4}{(2!)^2} + \dots \right]. \quad (6)$$

Inserting (6) into the boundary condition (3), we obtain an equation for the eigenvalues:

$$\begin{aligned} & \mu \left\{ 1 - \left[ \frac{2-\mu}{2} + \frac{(2-\mu)(6-\mu)\mu}{4 \cdot (2!)^2} + \frac{6(2-\mu)(6-\mu)(10-\mu)\mu^2}{4^3 \cdot (3!)^2} + \dots \right] \right. \\ & \left. \times \left[ 1 + \frac{(2-\mu)\mu}{4} + \frac{(2-\mu)(6-\mu)\mu^2}{16 \cdot (2!)^2} + \frac{(2-\mu)(6-\mu)(10-\mu)\mu^3}{64 \cdot (3!)^2} + \dots \right]^{-1} \right\} = \text{Bi}. \end{aligned} \quad (7)$$

The series in (6) and (7) are known to be convergent. Only the first twenty terms were used in the computer calculation. The contribution of the remaining terms was insignificant. For the beginning, the first three roots of the characteristic equation (7) were thus determined. As the starting point, the authors used the roots corresponding to a boundary condition of the first kind, i. e., to  $\text{Bi} \rightarrow \infty$  ( $\mu_3 = 10.6734$ ,  $\mu_2 = 6.6790$ , and  $\mu_1 = 2.7044$ ). The procedure for calculating  $\mu_n$  was based on a discrete decrement of  $\mu$

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in (7) in  $\Delta\mu = 0.0001$  steps. As the target value of  $\mu_n$  was chosen that which would convert Eq. (7) into an identity at a given value of the Biot number.

The eigenfunctions  $\psi_1$ ,  $\psi_2$ , and  $\psi_3$  were then calculated according to Eq. (6). Coefficients  $A_n$  of the series must satisfy the boundary condition at the channel entrance:

$$\sum_{n=1}^{\infty} A_n \psi_n = 1.$$

The constant coefficients  $A_n$  are found with the aid of this last expression, taking into account the orthogonality of eigenfunctions  $\psi_n$ .

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CALCULATION OF THE BOUNDARY LAYER AT A  
THIN PLATE HEATED SIMULTANEOUSLY BY  
RADIATION AND CONVECTION

V. V. Ivanov and I. L. Dunin

UDC 532.526:537.312.51

The authors analyze the compound heat transfer at a surface with a boundary layer heated from the other side simultaneously by thermal radiation and convection. The analysis is based on a laminar boundary layer and a thermally thin wall.

Applying to the original system of heat transfer equations

$$u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} = a \frac{\partial^2 \theta}{\partial y^2},$$

$$-\frac{\partial \theta}{\partial y} = \frac{\varepsilon \sigma_0 T_c^3}{\lambda} [K(1-\theta) + 1 - \theta^4] \text{ at } y = 0, \quad (1)$$

$$\theta = \theta_\infty \text{ at } y \rightarrow \infty, \quad (2)$$

$$0 < \theta_\infty = \frac{T_\infty}{T_c} < \theta = \frac{T(x, y)}{T_c} < 1, K = \frac{\alpha}{\varepsilon \sigma_0 T_c^3}, \quad (3)$$

the nonlinear integral transformation

$$W = \exp \left[ -\rho \int_0^\theta \frac{d\theta}{K(1-\theta) + 1 - \theta^4} \right] = \exp [-\rho f(K, \theta)] \quad (4)$$

leads to a problem where the boundary condition (2) is linear:

$$u \frac{\partial W}{\partial x} + v \frac{\partial W}{\partial y} = a \left[ \frac{\partial^2 W}{\partial y^2} + F(x, y) \right], \quad (5)$$

$$F(x, y) = \rho W \left[ \frac{\partial \theta / \partial y}{K(1-\theta) + 1 - \theta^4} \right]^2 (K + 4\theta^3 - \rho), \quad (6)$$

$$\frac{\partial W}{\partial y} = \rho \frac{\varepsilon \sigma_0 T_c^3}{\lambda} W \text{ at } y = 0, \quad (7)$$

$$W = \exp -\rho f(K, \theta_\infty) = W_\infty \text{ at } y \rightarrow \infty. \quad (8)$$

Group (6), which appears in the transformed energy equation (5), is minimized by means of parameter  $\rho$ . The condition that  $F(x, y) \rightarrow 0$  is obviously satisfied when  $\rho \rightarrow K + 4\theta^3$ . The unknown temperature  $\theta = \theta(x, y)$  is then found from relation (4) and the known solution to the linearized problem for  $W = W(x, y)$ .

The accuracy of the final results is evaluated by a bilateral error estimation, which also reveals the necessary accuracy criteria of the calculations.

The usefulness of this method is illustrated on several examples. A comparison of the calculated surface temperature with numerical values obtained by E. Sparrow and S. Lin confirms that the proposed method is highly accurate and reliable. It is also established that, within the given range, the values here do not differ from computer results by more than 1%.

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EFFECT OF AXIAL HEAT CONDUCTION THROUGH A  
PIPE ON THE TEMPERATURE OF THE PIPE WALL  
AND OF THE GAS STREAM INSIDE IT

G. G. Matlin

UDC 536.24:621.365

In the one-dimensional formulation of the problem, the temperature of a gas stream in a pipe  $t$  and the temperature of the heat dissipating pipe wall  $\tau$  are calculated, taking into account axial heat conduction through the pipe wall. The thermophysical parameters are assumed constant and axial heat conduction through the gas is disregarded. The cross section area of the wall  $f$ , the heat dissipation perimeter  $p$ , and the wetted perimeter  $p_w$  are assumed constant, also the initial heat dissipation per unit length  $q$  and the initial coefficient of heat transfer from wall to gas  $\alpha$ .

The problem reduces to the following equation with two dimensionless parameters  $K$  and  $L$ :

$$\frac{d^2\theta}{d\xi^2} + L \frac{d\theta}{d\xi} - K\theta = -K, \quad (1)$$

where

$$\xi = \frac{x}{l}, \quad \theta = \frac{(\tau - t)\alpha p}{q}, \quad K = \frac{\alpha p l^2}{\lambda f}, \quad L = \frac{\alpha p l}{c_p G} = 4St \frac{p}{p_w} \cdot \frac{l}{d_h}.$$

Parameter  $K$  characterizes the combined effect of convective heat transfer and axial heat conduction; parameter  $L$  is proportional to the pipe length referred to the pipe diameter  $d_h$ .

With  $\theta$  known,  $t$  and  $\tau$  are determined from the equation

$$t = t_0 + \frac{q l}{c_p G} \int_0^\xi \theta(\xi) d\xi, \quad \tau = t + \frac{\theta q}{\alpha p}.$$

An equation analogous to Eq. (1) is also given for the dimensionless wall temperature  $\bar{\tau} = (\tau - t_a)c_p G / q l$ ; like Eq. (1), this equation too involves the parameters  $K$  and  $L$ .

The solutions to these equations depend on  $K$  and  $L$ , but also on the dimensionless quantities which appear in the boundary conditions. For illustration, the problem is solved with either the thermal fluxes across the wall ends and the conditions of heat transfer between the end surfaces and the gas stipulated, or with the wall temperature at the pipe ends  $\tau_0$  and  $\tau_1$  given while  $t_0 = t_a$ . Calculations show that, if the effect of axial heat conduction is negligible (when  $K \rightarrow \infty$ ),  $\theta = 1$  within  $0 < \xi < 1$ . The effect of heat conduction on  $\theta$  becomes greater, generally, as  $K$  decreases and  $L$  increases. It is to be noted, however, that at given values of  $K$  and  $L$  there are such values of the boundary parameters which will make  $\theta = 1$  along the entire pipe (in the second case, for example, one can find these values by letting  $\bar{\tau}_0 = 1/L$  and  $\bar{\tau}_1 = 1 + 1/L$ ).

At sufficiently high values of  $K$ ,  $\theta = 1$  accurately enough within some middle segment of the pipe; as  $K$  increases, this segment becomes longer and at the limit,  $K \rightarrow \infty$ , it extends over the entire pipe except, perhaps, its ends. Within this segment  $\tau - t = \text{const} = q/\alpha p$  while the values of  $\bar{\tau}$  and  $\bar{t}$  differ by the constant term  $\delta\bar{t} = L/K$  from those calculated without accounting for heat conduction. Pipe segments where  $\theta \neq 1$  are zones under an appreciable influence of end effects.

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Equation (1) is easily extended to the case where  $q$  and  $\alpha$  are known functions of  $\xi$ . A general solution to this equation is obtained for  $\alpha = \text{const.}$

#### NOTATION

$l$	is the length of pipe;
$x$	is the space coordinate measured from the pipe entrance along the stream;
$d_h$	is the hydraulic diameter of pipe;
$G$	is the gas flow rate;
$St$	is the Stanton number;
$\lambda$	is the thermal conductivity of pipe wall;
$c_p$	is the specific heat of gas;
$t_0$	is the gas temperature before entering the pipe;
$t_a$	is the gas temperature at the pipe entrance.

TRANSIENT TEMPERATURE FIELD IN AN ANISOTROPIC  
PLATE UNDER A GIVEN THERMAL FLUX AT THE EDGE

Yu. M. Kolyano and V. I. Gromovyk

UDC 536.12+539.376

A semiinfinite anisotropic plate is considered at the edge of which a given thermal flux varies as a function of the space coordinate and of time  $q(y, \tau)$ . The initial temperature of the plate is zero. A general solution to the problem of transient heat conduction is obtained for this plate by means of integral Fourier transformations with respect to coordinate  $y$  and the Laplace transformation with respect to time  $\tau$ .

For the case where the thermal flux at the plate edge is given as

$$q(y, \tau) = q_0 S_+(\tau) \sum_{n=0}^N [S(y_-) - S(y_+)],$$

with  $S(y)$  and  $S_+(\tau)$  denoting the symmetric and the antisymmetric unit functions respectively and with  $N = 0, \pm 1, \pm 2, \dots, \pm y = y - nc - (2n \pm 1)d$ , the following expression is obtained for the transient temperature field in the plate:

$$T = \frac{Q}{2\pi\sqrt{k_y - k_{xy}^2}} \sum_{n=0}^N \int_{y_+ - k_{xy}x}^{y_- - k_{xy}x} K_0(\rho, \omega) d\xi.$$

Here

$$K_0(\rho, \omega) = \int_0^{\omega} \exp \left[ -\frac{\rho}{2} \left( \omega + \frac{1}{\omega} \right) \right] \frac{d\omega}{\omega}; \quad \rho = \kappa \sqrt{\frac{\xi^2}{k_y - k_{xy}^2} + x^2};$$

$$\omega = 2a\kappa\tau \sqrt{\frac{k_y - k_{xy}^2}{\xi^2 + x^2(k_y - k_{xy}^2)}}; \quad Q = \frac{q_0}{\lambda_{11}}; \quad k_{xy} = \frac{\lambda_{12}}{\lambda_{11}}; \quad k_y = \frac{\lambda_{22}}{\lambda_{11}};$$

$\kappa^2 = \alpha_z / \lambda_{11} \delta$ , with  $\alpha_z$ ,  $\lambda_{ij}$ , and  $a$  denoting the heat transfer coefficient at the lateral plate surfaces ( $z = \pm \delta$ ), the thermal conductivity, and the thermal diffusivity respectively;  $x, y$  are the space coordinates of the plate and  $c$  is the distance between segments of the edge surface  $x = 0$ , at which the thermal flux has been specified.

The obtained solution is valid for an infinitely large anisotropic plate with an array of equidistant slots, length  $2d$ , at the edges of which the thermal flux specified at time  $t = 0$  remains constant thereafter.

For a plate of grade KAST-V glass-Textolite with a single slot, it is explained how the degree of orthotropy and the heat transfer from the lateral surfaces  $z = \pm \delta$  to the ambient medium affect the steady-state temperature field, and how the degree of orthotropy affects the transient temperature field in a thermally insulated plate. The numerical results, presented graphically, indicate that the temperature in a thermally insulated plate rises with time. Simultaneously, also the relative effect of orthotropy in the material increases. The steady-state temperature field decreases as the rate of heat transfer from the lateral surfaces rises. The relative effect of orthotropy also decreases then.

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APPROXIMATE METHOD OF CALCULATING TRANSIENT  
THERMAL PROCESSES IN A MULTILAYER MEDIUM

N. P. Gaponenko

UDC 536.24

An approximate method is proposed for calculating transient thermal processes in a multilayer wall with a known law of power variation at one surface. The calculation of the temperature field in such structures can be simplified, if one assumes that during a step change in power:

1. the layers farther removed from the power source than a given layer do not affect the stabilization of the temperature difference between the beginning and the end of any segment of the multilayer structure;
2. when the temperature difference between the beginning and the end of a given layer has not yet reached its maximum possible magnitude, at every point in the layers preceding the given one the temperature rises by an amount equal to the temperature difference between the beginning and the end of that given layer.

The first assumption makes it possible to assign a zero temperature at the boundary of the given region; the second assumption allows the differential equation for the preceding layers to be averaged over their respective thicknesses. As a result, the processes of heat transfer in the  $i$ -th layer are described by a self-contained system of differential equations:

$$\frac{\partial^2 \theta_i}{\partial x^2} - \frac{1}{\alpha_i^2} \frac{\partial \theta_i}{\partial t} = 0;$$
$$\frac{\partial \theta_i}{\partial x} = -\frac{q}{\lambda_i} + M_i \frac{\partial \theta_i}{\partial t} \quad (x = 0);$$
$$\theta_i = 0 \quad (x = d_i),$$

and the temperature drop across the multilayer wall is represented by the sum of temperature drops across all layers.

The method was proved out by simulating transient thermal processes on RC-networks. For comparing test results with calculations, expressions have been derived which relate the nominal values of the RC elements to the parameters of a multilayer wall and to the criterial parameters appearing in the solutions to the self-contained system of differential equations.

This method is useful for calculating thermal processes which occur under any law of power generation in a multilayer wall.

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# ANALYSIS OF HEAT CONDUCTION IN MULTILAYER SYSTEMS

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and R. V. Pal'tsun

UDC 536.21

The article deals with a multilayer system (any number of layers). It is assumed that the layers in this system have each a uniform thickness. Each layer has a system of coordinates assigned to it, with the origin on the upper boundary surface and with a common normal axis. The layers are numbered consecutively from top to bottom.

Some special cases of this problem had been analyzed earlier [1, 2].

The temperature field in any layer is defined by a function which is the solution to a harmonic equation. Applying to this equation the complex Fourier transformation with respect to variable  $x$  (the  $x$ -axis runs along the layer boundary) yields an ordinary second-order differential equation with constant coefficients with respect to the transform of the sought function. The solution to this equation depends on two arbitrary coefficients, and for the entire multilayer system on  $2n$  coefficients ( $n$  is the number of layers) which must be determined from the boundary conditions and from the continuity condition regarding temperatures as well as thermal fluxes at interlayer boundaries. The continuity condition yields recurrence formulas for the unknown coefficients, which makes it necessary to determine only the coefficients  $A_1$  and  $B_1$  for the first layer. In fundamental boundary-value problems of heat conduction in a multilayer system, the determination of  $A_1$ ,  $B_1$  and thus also of the remaining coefficients, by virtue of the recurrence relation, reduces to solving just one linear algebraic equation regardless of the number of layers in the system. Thus, for example, with the temperature given at the upper and the lower boundary of the system

$$T_1(x, 0) = f(x); T_n(x, -h_n) = 0, \quad (1)$$

the use of formulas here where  $A_n$ ,  $B_n$  are expressed in terms of  $A_1$ ,  $B_1$  will yield  $A_1 = H_n(\xi, h_1, h_2, \dots, h_n)B_1$  with  $h_n$  denoting the thicknesses of respective layers and function  $H_n$  characterizing the thermophysical properties of the system. The properties of function  $H_n$  are established. In this way, in order to determine the temperature field in the multilayer system, it is necessary only to use the theorem of the inverse Fourier transformation.

The proposed method is illustrated on two problems with mixed boundary conditions, the solution of which reduces to the solution of double and triple integral equations respectively [4].

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A PROBLEM OF HEAT CONDUCTION IN A  
SEMIINFINITE HOLLOW CYLINDER

G. M. Bartenev, A. I. Zhornik,  
and É. M. Kartashov

UDC 536.2.01

The problem of the temperature field  $T(r, z, t)$  in a semiinfinite hollow cylinder  $r_1 < r < R$  is solved for the most general initial and boundary conditions, encompassing the entire class of special problems within this range.

Mathematically the problem reduces to solving the two-dimensional equation of heat conduction

$$\frac{\partial T}{\partial t} = k \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} \right), \quad r_1 < r < R, \quad z > 0, \quad t > 0 \quad (1)$$

for the following initial and boundary conditions

$$T(r, z, t) \Big|_{t=0} = A(r, z), \quad (2)$$

$$\alpha_1 \frac{\partial T}{\partial r} \Big|_{r=r_1} - \alpha_2 T \Big|_{r=r_1} = \alpha_3 C(z, t), \quad (3)$$

$$\alpha_4 \frac{\partial T}{\partial r} \Big|_{r=R} + \alpha_5 T \Big|_{r=R} = \alpha_6 \Theta(z, t), \quad (4)$$

$$-\alpha_7 \frac{\partial T}{\partial z} \Big|_{z=0} + \alpha_8 T \Big|_{z=0} = \alpha_9 B(r, t), \quad (5)$$

with  $\alpha_i \geq 0$  - const and functions  $A(r, z)$ ,  $C(z, t)$ ,  $\Theta(z, t)$ ,  $B(r, t)$  given.

The solution to problem (1)-(5) is sought in the class of functions to which the two-dimensional Laplace transformation with respect to variables  $z, t$  is applicable and yields

$$\begin{aligned} T(r, z, t) = & \sum_{n=1}^{\infty} \frac{\pi^2 \gamma_n^2}{2} \cdot \frac{J^2(\gamma_n R)}{(\alpha_5^2 + \alpha_4^2 \gamma_n^2) J^2(\gamma_n r_1) - (\alpha_2^2 + \alpha_1^2 \gamma_n^2) J^2(\gamma_n R)} \\ & \times \left\{ \frac{k}{\pi} \int_0^t \exp[-k\gamma_n^2(t-\tau)] d\tau \int_0^{\infty} \eta(z, \xi, k(t-\tau)) \left[ \alpha_3 C(z, \tau) \right. \right. \\ & \left. \left. - \alpha_6 \frac{Y(\gamma_n r_1)}{Y(\gamma_n R)} \Theta(\xi, \tau) \right] d\xi - \frac{1}{2} \exp[-k\gamma_n^2 t] \int_0^{\infty} \eta(z, \xi, kt) d\xi \right. \\ & \left. \times \int_{r_1}^R \tilde{\alpha} A(\tilde{\alpha}, \xi) U_0(\gamma_n \tilde{\alpha}) d\tilde{\alpha} - \frac{\alpha_9}{2\alpha_7} k \int_{r_1}^R \tilde{\alpha} U_0(\gamma_n \tilde{\alpha}) d\tilde{\alpha} \right. \\ & \left. \times \int_0^t \exp[-k\gamma_n^2(t-\tau)] \eta(z, 0, k(t-\tau)) B(\tilde{\alpha}, \tau) d\tau \right\} U_0(\gamma_n r), \quad (6) \\ \eta(z, \xi, t) = & \chi(z + \xi, t) - \chi(z - \xi, t) - 2 \frac{\alpha_7}{\alpha_8} \left[ \sqrt{\frac{\alpha_8^2}{\pi t} \frac{\alpha_7^2}{\alpha_7^2}} \exp\left[-\frac{(z + \xi)^2}{4t}\right] \right. \\ & \left. - \frac{\alpha_8^2}{\alpha_7^2} \exp\left[(z + \xi) \frac{\alpha_8}{\alpha_7} + \frac{\alpha_8^2}{\alpha_7^2} t\right] \operatorname{erfc}\left(\frac{z + \xi}{2\sqrt{t}} + \frac{\alpha_8}{\alpha_7} \sqrt{t}\right) \right], \end{aligned}$$

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$$\chi(z, t) = \sqrt{\frac{1}{\pi t}} \exp\left(-\frac{z^2}{4t}\right),$$

where  $\gamma_n$  are the roots of the equation

$$\alpha_5 U_0(\gamma R) - \gamma \alpha_4 U_1(\gamma R) = 0,$$

and  $U_1(\gamma R)$ ,  $U_0(\gamma R)$ ,  $U_0(\gamma_n r)$ ,  $J(\gamma_n R)$ ,  $J(\gamma_n r)$ ,  $Y(\gamma_n R)$ ,  $Y(\gamma_n r)$  is a combination of first- and second-order, first- and second-kind Bessel functions of a real argument.

The obtained solution (6) covers the entire gamut of special solutions corresponding to specific values of coefficients  $\alpha_i$ .

NONUNIFORM MOTION OF A PARTICLE IN  
A HYDROCYCLONE

Yu. N. Boldyrev

UDC 541.123.012.5

For the design of a hydrocyclone it is important to know the velocity of a particle moving radially toward the apparatus wall. This velocity determines the dwell time of a particle in the apparatus and the productivity of the latter. During the motion of a particle relative to the apparatus wall there act on this particle centrifugal and Coriolis forces of inertia, gravitational forces, forces due to the flow of liquid toward the axis and the drain tube, and lift and drag forces. Inasmuch as most of these forces are functions of the particle coordinates, the particle will move through the liquid in a nonuniform manner. This requires that the inertia term be taken into account in the equation of relative motion. The nonuniformity of particle motion produces additional perturbations in the liquid, which are reflected in changes in its kinetic energy. As a consequence, the particle encounters an additional drag which can be calculated by the method of coupled masses.

By a projection of all these forces acting on a particle on the hydrocyclone radius, a differential equation is obtained which describes the relative radial motion of a particle and which reduces to the Bernoulli equation, if the radial and the vertical velocity components of the counterflowing liquid are only weak functions of the radius of revolution and if  $\omega = \text{const}$ .

The solution to the resulting equation is represented as

$$\text{Re}^2 = \frac{4}{3} \text{ArFr} \left( \frac{1}{\xi_0} + \frac{C}{\xi_0^2} + \frac{C}{\xi_0^2 e^{2a(r-r_d)}} - \frac{D}{\xi_0} \right) - \frac{4}{3} \text{Ar} \left( \frac{E}{\xi_0} - \frac{F}{\xi_0^2} + \frac{F}{\xi_0^2 e^{2a(r-r_d)}} - \frac{K}{\xi_0} \right),$$

with the Reynolds number  $\text{Re} = u_r d / \nu$ , the Archimedes number  $\text{Ar} = g d^3 \Delta / \rho_0 \nu^2$ , the Froude number  $\text{Fr} = \omega^2 r / g$ ,  $a = (3/4) \xi_0 (\rho_0 / d (\rho + 1/2 \rho_0))$ ,

$$C = \frac{4d \left( \rho + \frac{1}{2} \rho_0 \right)}{6\rho_0 r}; \quad D = \frac{r d}{r e^{2a(r-r_d)}}; \quad E = \frac{A \left( \rho + \frac{1}{2} \rho_0 \right)}{g \Delta};$$

$$F = \frac{4Ad \left( \rho + \frac{1}{2} \rho_0 \right)}{6r \Delta \rho_0 g}; \quad K = DE;$$

$$A = \frac{3\xi_1 \rho_0 v_R^2 \sin \alpha}{4d \left( \rho + \frac{1}{2} \rho_0 \right)} + \frac{3\xi_2 \rho_0 v_0^2 \cos \alpha}{4d \left( \rho + \frac{1}{2} \rho_0 \right)}; \quad B = 1 - \frac{A}{\frac{\Delta \omega^2 r}{\rho + 1/2 \rho_0}};$$

$v_R$  and  $v_\omega$  are the particle velocities along and perpendicular to the hydrocyclone generatrix. These velocities are assumed equal to the respective velocities of the liquid, which should be given. To the projections of these velocities of the radii correspond drag coefficients  $\xi_1$ ,  $\xi_2$  which are determined according to the Reynolds number for a particle,  $\xi_0$  is a drag coefficient,  $d$  is the particle diameter,  $\alpha$  is half the apex angle of the hydrocyclone cone,  $\rho$  is the particle density,  $\rho_0$  is the liquid density,  $u_r$  is the relative radial velocity of a particle ( $u_r = u_p - u_L$ ),  $u_p$  is the absolute velocity of a particle,  $u_L$  is the liquid velocity,  $\nu$  is the liquid viscosity,  $\Delta = \rho - \rho_0$  is the density difference between particle and liquid,  $\omega$  is the

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angular velocity of a revolving particle,  $r$  is the radius of a particle revolution, and  $r_d$  is the radius of the drain tube.

Calculations based on test data have shown that the inertia term in the differential equation of motion must be included in the design of a hydrocyclone for separating coarse suspensions, where a solid particle moves in a vortex of liquid.

CALCULATING THE EFFICIENCY OF DUST COLLECTORS  
FOR TRAPPING HYDROPHILIC AND HYDROPHOBIC  
PARTICLES WITH LIQUID SPRAYS

R. A. Burtseva and A. T. Litvinov

UDC 66.011+621.928.9

A method is proposed for calculating the trap factor and the efficiency of dust collectors for trapping hydrophilic and hydrophobic particles with liquid sprays.

In the equations proposed earlier by various authors for calculating the trap factor and the trapping efficiency no account is taken of the wettability of particles, but an empirical coefficient appears which is based on a preliminary determination for a definite dispersion spectrum and operating conditions. For this reason, calculated values differ appreciably from measured values, especially in the case of hydrophobic particles.

Here an equation is shown which yields a satisfactory agreement between experimental and theoretical values for the trap factor:

$$\varepsilon = \frac{V_0}{V_0 + KLi}, \quad (1)$$

where

$$Li = \frac{V_0(l_{ig} - l_{if}) + Kl_{ig}}{Kl_{if}} \quad (2)$$

is a hydrophobic parameter which characterizes particles of trapped dust. The trapping efficiency is calculated according to the formula

$$\eta = \sum_{i=1}^m \varepsilon_i q_i n_i. \quad (3)$$

The inertial path of hydrophilic particles is calculated according to the following equations:

a) for a Reynolds number in the range  $Re \leq 0.2$

$$l_{ig} = \tau V_0, \quad (4)$$

with  $\tau = d^2 \rho_2 / 18 \mu$ ;

b) for a Reynolds number in the range  $Re \leq 300$

$$l_{ig} = \gamma \ln \frac{\alpha V_0 + \beta}{\alpha V_i + \beta}, \quad (5)$$

with  $\alpha = 0.75 A \rho_1 / d \rho_2$ ,  $\beta = 0.75 B \mu / d^2 \rho_2$ , and  $\gamma = 4d \rho_2 / 3A \rho_1$ .

The inertial path of hydrophobic particles is calculated according to the following equations:

a) for a Reynolds number in the range  $Re \leq 0.2$

$$l_{if} = \tau (V_0 - C), \quad (6)$$

with  $C = (8\sigma / d \rho_2)^{0.5}$ ;

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b) for a Reynolds number in the range  $Re \leq 300$

$$l_{if} = \gamma \ln \frac{\alpha V_0 + \beta}{\alpha C + \beta}. \quad (7)$$

The average diameter of liquid droplets in pneumatic and centrifugal atomizers is calculated according to the formula

$$D_m = \frac{\sum_{i=1}^m D_i^4 n_i}{\sum_{i=1}^m D_i^3 n_i}. \quad (8)$$

The values of the trap factor and of the trapping efficiency calculated for hydrophilic and hydrophobic particles yield a satisfactory agreement with these and other authors' test data. The error does not exceed 0.05-0.80.

The effect of particle wettability on the trapping efficiency is also discussed.

#### NOTATION

$\varepsilon, \varepsilon_i$	are the trap factors referred to particles in a single drop and to the $i$ -th diameter;
$V_0, V_i$	are the relative velocity between particle and droplet, and referred to the $i$ -th distance from the droplet, m/sec;
$d$	is the particle diameter, m;
$D_m, D_i$	are the average diameter of droplets, and referred to the $i$ -th dimension, m;
$A, B$	are the constants equal to 0.12 and 37 respectively for $Re \leq 300$ ;
$\rho_1, \rho_2$	are the density of medium and of particle respectively, $kg/m^3$ ;
$\mu$	is the viscosity of medium, $N \cdot sec/m^2$ ;
$l_{if}, l_{ig}$	are the inertial path of hydrophobic and hydrophilic particles respectively, m;
$\eta$	is the trapping efficiency;
$q_i, n_i$	are the relative quantity of particles and droplets of the $i$ -th fraction;
$m$	is the number of particle and droplet fractions;
$\sigma$	is the surface tension of the liquid, N/m;
$K$	is the coefficient equal to $10.62 \mu D_m / d^2 \rho_2$ for $Stc > 0.25$ ;
$Stc = V_0 d^2 \rho_2 / 18 \mu D_m$	is the Stokes number.



MASS TRANSFER DURING LIXIVIATION OF THE  
SOLID PHASE FROM POROUS BODIES

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UDC 66.015.23

The article deals with the kinetics of extracting the solid phase from a porous spherical particle in which this solid phase forms spherical inclusions uniformly and continuously distributed over the entire volume.

When the porous particle is immersed in a solvent, all pores become filled with liquid and there follows a diffusion of solute molecules into the particle volume with a resulting loss of weight  $g$  by an individual inclusion in accordance with the relation [2]:

$$-\frac{\partial g}{\partial t} = 4\pi\rho D (c_s - c^*), \quad \rho = \sqrt[3]{\frac{3g}{4\pi\gamma}} \quad (1)$$

The soluble inclusions are, therefore, sources of diffusing substance with an intensity  $-N(\partial g/\partial t)$  and, consequently, the following equation applies to diffusion with spherical symmetry:

$$\frac{\partial c^*}{\partial t} = D \left( \frac{\partial^2 c^*}{\partial r^2} + \frac{2}{r} \cdot \frac{\partial c^*}{\partial r} \right) - \frac{N}{m} \cdot \frac{\partial g}{\partial t} \quad (2)$$

The constraints on the unknown quantities  $c^* = c^*(r, t)$  and  $g = g(r, t)$  will be

$$c^*(r, 0) = 0; \quad c^*(r, \infty) = 0; \quad c^*(R, t) = 0; \quad \left( \frac{\partial c}{\partial r} \right)_{r=0} = 0; \\ g(r, 0) = g_0; \quad g(r, \infty) = 0; \quad \left( \frac{\partial g}{\partial r} \right)_{r=0} = 0. \quad (3)$$

When the porous particle contains much less soluble substance than necessary to saturate its pore volume, i. e., when  $c^* \ll c_s$ , then the system of equations (1) and (2) becomes linear and under constraints (3) we obtain the solution

$$\frac{u}{\eta\gamma\rho} = (1 - \beta\tau)^{\frac{3}{2}} + 0,1 \sqrt{1 - \beta\tau} - \frac{9\beta}{\pi^4} \sum_{k=1}^{\infty} \frac{1}{k^4} \exp(-k^2\pi^2\tau) \\ + \frac{9\beta^{\frac{3}{2}}}{2\pi^{9/2}} \sum_{k=1}^{\infty} \frac{1}{k^5} \exp\left(\frac{k^2\tau^2}{\beta} - k^2\pi^2\tau\right) \left[ \operatorname{erf}\left(\frac{k\pi}{\sqrt{\beta}}\right) - \operatorname{erf}\left(k\pi \sqrt{\frac{1}{\beta} - \tau}\right) \right] \quad (4)$$

At high concentrations of the substance to be extracted there form two regions within a particle [3]: an outer region without sources of extractable substance and an inner region, within a radius  $r_0$ , containing the soluble solid substance. Assuming the concentration of distribution to be quasisteady in the outer region, and applying the integral relation in [3], we obtain the solution to Eqs. (1) and (2) in the form

$$\frac{u}{\eta\gamma\rho} = \varphi_0^3 - \psi \left( \varphi_0^3 - \frac{\varphi_0^2}{2} - \frac{\varphi_0}{2} \right), \quad (5)$$

$$\tau = \left( \frac{1}{\psi} - 1 \right) \left( \frac{1}{6} - \frac{\varphi_0^2}{2} + \frac{\varphi_0^3}{3} \right) - \frac{1}{6} (\ln \varphi_0 + \varphi_0 - \varphi_0^2). \quad (6)$$

These results represent an extension and a refinement of the kinetic equations derived earlier [1, 2].

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## NOTATION

$c^*$	is the concentration of extractable substance in the liquid phase of particles, $\text{kg}/\text{m}^3$ ;
$g_0$	is the initial weight of a single inclusion, kg;
$g$	is the weight of an inclusion at any instant of time, kg;
$\rho$	radius of an inclusion, m;
$\gamma$	is the density of the inclusion substance, $\text{kg}/\text{m}^3$ ;
$D$	is the diffusivity, $\text{m}^2/\text{sec}$ ;
$R$	is the radius of a porous particle, m;
$N$	is the number of inclusions per unit particle volume, $1/\text{m}^3$ ;
$m$	is the porosity of original specimen, $\text{m}^3/\text{m}^3$ ;
$t$	is the time, sec;
$r_0$	is the radius of the region containing the extractable solid substance, m;
$u$	is the total mass of extractable substance in a particle, $\text{kg}/\text{m}^3$ ;
$\eta$	is the weight fraction of inclusions in the final specimen;
$\gamma_p$	is the density of specimen, $\text{kg}/\text{m}^3$ ;
$c_s$	is the saturation concentration of the solution, $\text{kg}/\text{m}^3$ ;
$\tau = Dt/R^2$	is the dimensionless time;
$\varphi = r/R$	is the dimensionless radius;
$\beta = 2R^2 c_s / \rho_0^2 \gamma$ ;	
$\psi = mc_s / \eta \gamma_p$	are the characteristic dimensionless parameters.

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## STUDY OF GAS FLOW IN PARALLEL JETS

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UDC 533.5.72

The flow pattern in supersonic jets discharging from parallel nozzles was studied for a qualitative evaluation. The inner wave structure of the flow is now described here along with the gas leakage characteristics in various planes. On the basis of test results, an approximate method is proposed for calculating the parameters in the inner regions of multinozzle jets.

The experiments were performed in a supersonic gasodynamic tunnel and covered the following ranges of parameter values: Mach number at the nozzle throat  $M_a = 1.0-2.5$ , inefficiency of jet discharge  $\eta = 0.6-31.4$ , divergence angle of nozzle cone  $\alpha = 5-20^\circ$ .

The following procedures and instruments were used for this experimental study. Shadow photographs were obtained with the model IAB-451 optical instrument. The total and the static pressures within the inside regions of a jet were measured by means of separate venturis in conjunction with class 0.5 manometers. The physical nature of the gas flow was studied on a "semimodel" test stand by, essentially, replacing the interaction between parallel jets with the interaction between a single jet and a plane solid wall. In the course of this study, the pattern of gas flow was observed by visual means using various viscous tracer substances.

An analysis of the experiments leads to the conclusion that the gas flow in the "interaction" plane passing between jets is analogous to a discharge of plane jets: the tracer substance left on that plane vestiges of a wave structure inherent to a jet. The optical patterns indicate also the region of jet separation from the barrier.

The substitution of a solid wall for the interaction plane in a four-nozzle jet has made it possible to measure the static pressure in the given plane. For this purpose, inductive DD10 probes were attached at the drain holes in the metal plate.

According to the results of measurements, the first pressure peak corresponds to the location of an oblique compression jump in the jet-plate interaction region. There follows a rapid expansion all the way to the region where the jet separates. The second pressure peak is due to a compression jump in the region where the boundary layer separates.

An important item in the study of multinozzle jets was the variation in the Mach number along the block axis. For the purpose of determining it, the total and the static pressures were measured at various points along the four-nozzle jet axis. The analyzed data indicated a linear variation along such a multinozzle jet.

On the basis of this last test result and earlier conclusions concerning the two-dimensional mode of gas flow in the interaction plane, an approximate method could be proposed for calculating the gasodynamic parameters in the interaction plane.

The calculations are to be performed in the following sequence: from the known values of parameters at the nozzle throat one determines the Mach number at the jet boundary and its inclination angle to the axis; then the interaction parameters of the mixing jets are calculated by the formulas for an oblique compression jump; then the two-dimensional jets in the interaction plane are calculated; then the Mach number before a straight jump in a component jet is calculated; finally, a straight line of the Mach number variation along the jet axis is plotted.

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The equations of a polytropic process with a variable mass of ideal gas are derived as follows. In the Gibbs equation  $dU = Tds - pdV + \varphi dG$  one substitutes  $ds = sdG + Gds$  and obtains  $dU = TGds - pdV + idG$ , but at the same time  $dU = udG + Gc_v dT$ . Equating  $dU$  in the last two expressions and inserting

$$dT = \frac{1}{R} d \left( \frac{pV}{G} \right) = T \left( \frac{dV}{V} + \frac{dp}{p} - \frac{dG}{G} \right)$$

one obtains, after integration, an equation of state which relates the generalized potential, i. e., the pressure to all generalized coordinates of the system [1]:

$$p \left( \frac{V}{G} \right)^\kappa \exp(-s/c_v) = \text{const.} \quad (1)$$

The equation of a polytropic process can be found by adding to expression (1) those which relate changes in entropy and in mass to changes in volume, and which ensure a constant polytropic exponent in the process  $GV^\gamma = G_1 V_1^\gamma = \text{const}$ ,  $V^\vartheta \exp(-s/c_v) = V_1^\vartheta \exp(-s_1/c_v) = \text{const}$ . As a result,

$$pV^{\kappa(1+\gamma)+\vartheta} = \text{const.} \quad (2)$$

The equation for the temperature in a polytropic process is

$$TV^{(\kappa-1)(1+\gamma)+\vartheta} = \text{const.} \quad (3)$$

When  $\vartheta = 0$  and  $\gamma = -1$ , Eqs. (2) and (3) describe the suction and the delivery process in theoretical cycles of displacement machines.

It is shown that in a polytropic process with a variable mass the specific heat remains constant. Integration of Eq. (2) yields expressions for the compression work  $L_{12} = \int_1^2 pdV$  and for the useful work  $L_T = \int_1^2 Vdp$ .

A method of calculating the efficiency of processes with a loss of active substance is also discussed.

#### NOTATION

U	is the internal energy of a system;
u	is the specific internal energy;
S	is the entropy;
s	is the specific entropy;
p	is the pressure;
V	is the volume;
$\varphi$	is the chemical potential;
G	is the mass;
i	is the specific enthalpy;
R	is the gas constant;
$c_v$	is the specific heat at constant volume;
$\kappa$	is the adiabatic exponent.

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